# MATH 565 Monte Carlo Methods

Fred J. Hickernell  Final Exam—Take Home Part  Due by 2 PM, December 11, 2025

*Instructions:*

i. *This is a **take-home** exam with **ONE** question(s). Attempt all. The maximum number of points is **35**.*

ii. *This take-home exam is due at the time listed above. There is no time limit other than the due date.*

iii. *You must turn in your files via Canvas and hard copies to the instructor.*

iv. *You may use AI tools (e.g., ChatGPT) and other inanimate resources. You may **not** get help from any human (classmates, tutors, friends, family, or online responders). All solutions must be your own work, and you are responsible for checking that AI-generated content is correct.*

v. *Show all your work to justify your answers. Answers without adequate justification will not receive credit.*

vi. *If you have clarification questions, email the instructor at* `hickernell@illinoistech.edu`*. Mathematical help will not be given.*

*I understand these instructions and have not relied on any help for this exam beyond what is allowed, nor provided any help to anyone else beyond what is allowed.*

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*Signature*  *Date*

1. (35 points) This problem concerns modeling the price of a financial option. We model the price of a stock in terms of a geometric Brownian motion, as

$$S(t_j, X_j) = S(0) \exp\bigl((r - \sigma^2/2)t_j + \sigma X_j\bigr),$$
$$t_j = jT/d, \quad (X_1, \ldots, X_d)^\top \sim \mathcal{N}(\mathbf{0}, \Sigma),\ \Sigma = \bigl(\min(t_j, t_k)\bigr)_{j,k=1}^d$$

We consider two kinds of option *payoffs* with the discount factor:

Asian arithmetic mean call: $\text{payoff}(\boldsymbol{X})$

$$= \max\left(\frac{1}{d}[S(0)/2 + S(t_1, X_1) + \cdots + S(t_{d-1}, X_{d-1}) + S(T, X_d)/2] - K, 0\right) \times \exp(-rT)$$

Lookback call: $\text{payoff}(\boldsymbol{X}) = \max\Bigl(S(T, X_d) - \min_{0 \le j \le d}\bigl(S(t_j, X_j)\bigr)\Bigr) \times \exp(-rT)$

The basic parameters that we will use are

$$\begin{aligned}
\text{time to maturity } T &= 1/4 \text{ (quarter of a year)} \\
\text{number of time steps } d &= 13 \text{ (weekly steps)} \\
\text{initial stock price } S(0) &= 50 \\
\text{interest rate } r &= 0.02 \\
\text{volatility } \sigma &= 0.40 \\
\text{strike price } K &= 50 \text{ (at the money)}
\end{aligned}$$

The option price is the expected value of the discounted payoff:

$$\text{price} = \mathbb{E}[\text{payoff}(\boldsymbol{X})].$$

a. (20 points) What prices do you get for the two different options using $n = 2^{17}$ IID samples, and what are your confidence intervals for the prices?

b. (8 points) What prices do you get for the two different options using $n = 2^{17}$ Sobol' samples, and what are your confidence interval for the prices? How do these confidence intervals compare to those for IID sampling?

c. (7 points) If you change your monitoring to daily ($d = 91$), how do the option prices change in part a., and why? How does the time required change? Explain these changes in prices and runtime.